H2SI – A New Perceptual Colour Space

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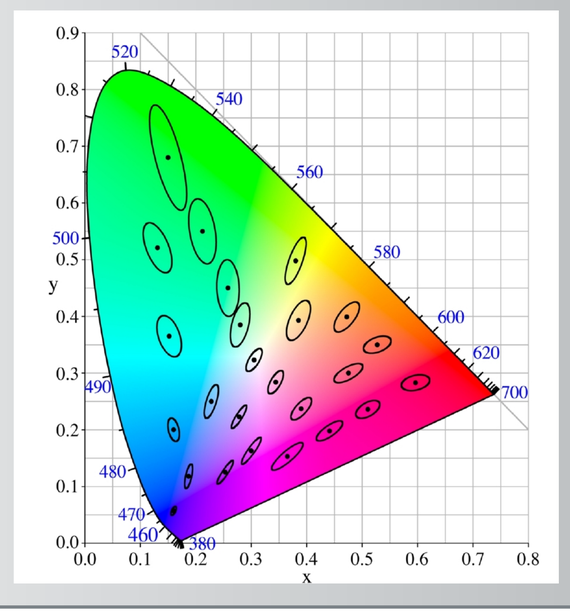
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INTRODUCTION

We introduce a new colour space whose definition is inspired by the formalism of quantum information theory. It is equipped with a metric that shares many properties with the human perception of colour, and we derive its most important geometric properties. The new colour space is well suited for algorithmic purposes, as we demonstrate with the 'Eigen' colour decomposition of images.

Even today colour remains a miracle. Its nature, representation, perception, and geometric properties have been studied for centuries, by scientists including Newton, Goethe, Young, Maxwell, Hering, Helmholtz, Munsell, Silberstein, and Schrödinger, to name but a few, and although much progress has been made, the 'holy grail' of colour science, a perceptually uniform (isotropic) colour space, has not yet been found. Isotropic means that perceived colour attributes map to a space where unit differences in one dimension are equal to unit differences in the other dimensions. In loose terms, these attributes are given by the hue, saturation and intensity (HSI) of a colour, though their exact definition varies greatly.

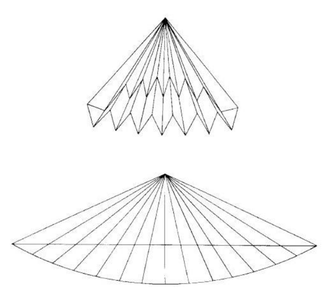
The most widely used explicit definition of a colour representation in a mathematical space is the XYZ colour matching functions space, which was defined by the CIE (Commission internationale de l'éclairage) in 1931. In the 1940s MacAdam showed that the CIE XYZ colour space is not isotropic [1]. Instead of circles of perceived unit differences of colour it contains ellipses in the xy chromaticity diagram which are elongated in chroma with respect to hue.



SUPER-IMPORTANCE OF HUE

MacAdam's results were strengthened by Judd [2], Schrödinger [3], and Silberstein [4], who concluded that no ideal colour space, especially no three-dimensional Euclidean space supports geometrical properties where MacAdam ellipses, i.e. perceived unit differences of colour, can lie on unit circles. Judd termed this property of colour perception *'Super-importance of hue differences'*.

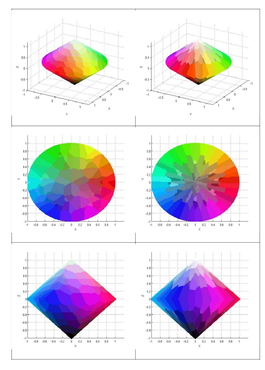
Judd pointed out that any mathematical model containing this property requires a space for colour representations that enables unit circles to have a circumference to radius ratio of roughly 4π, i.e. twice the circumference of a unit circle in Euclidean space. He visualized this as a 'fan-crinkled surface'.



THE HSI COLOUR SPACE

We use the HSI colour space, given by a double cone as introduced by Hering, as our basic colour space. Colour is defined in terms of three variables, namely hue, $0 \leq H < 2\pi$, saturation, $0 \leq S \leq 1$, and intensity, $0 \leq I \leq 1$. If the 'rgb' space (red, green, blue) is the starting point for colour representations, our preferred choice of transformation is:

$$\begin{aligned} x &= \min(r,g,b) \\ y &= \max(r,g,b) \\ I &= (x+y)/2 \\ S &= y-x \\ c_1 &= r-(g+b)/2 \\ c_2 &= \frac{\sqrt{3}(b-g)}{2} \\ H &= \arctan(c_2/c_1) \end{aligned}$$



THE H2SI COLOUR SPACE

We encode H, S, and I as qubit states, then define a state $|\psi\rangle$ in terms of them:

$$|I\rangle = \begin{pmatrix} \sqrt{I} \\ \sqrt{1-I} \end{pmatrix}, \quad |S\rangle = \begin{pmatrix} \sqrt{\frac{S}{2}} \\ \sqrt{1-\frac{S}{2}} \end{pmatrix}, \quad |H\rangle = \begin{pmatrix} \cos(2H)e^{-iH} \\ \sin(2H)e^{iH} \end{pmatrix}, \quad |\psi\rangle = \begin{pmatrix} S_1|H\rangle \\ S_2|I\rangle \end{pmatrix}$$

and we define the H2SI colour space as the four-dimensional space:

$$\begin{aligned} x_1 = \psi_3 &= \sqrt{I\left(1-\frac{S}{2}\right)} \\ x_2 = \psi_1 &= \sqrt{\frac{S}{2}}\cos(2H)e^{-iH} \\ x_3 = \psi_2 &= \sqrt{\frac{S}{2}}\sin(2H)e^{iH} \\ x_4 = \psi_4 &= \sqrt{(1-I)\left(1-\frac{S}{2}\right)} \end{aligned}$$

PROPERTIES AND GEOMETRY

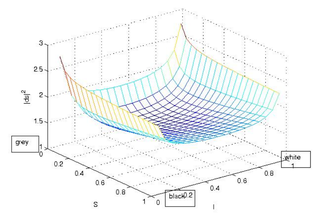
H2SI stands for 'Hilbert square, super-importance'. The variables x_1 and x_4 can be combined into a single complex variable:

$$\begin{aligned} X_1 = x_1 + ix_4 &= \sqrt{(1-\tfrac{S}{2})}\,e^{i\arctan\sqrt{\frac{1-I}{I}}} \\ X_2 &= x_2 \\ X_3 &= x_3 \end{aligned}$$

The line element is independent of H, and it contains no cross-terms, so the metric tensor is diagonal:

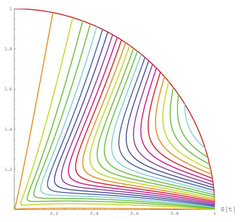
$$ds^2 = \sum_{i=1}^{4}|dx_i|^2 = \frac{dS^2}{4S(2-S)} + \frac{2-S}{8I(1-I)}dI^2 + \frac{5}{2}SdH^2$$

The ratio of the circumference of a fully saturated colour circle to its diameter is $4\sqrt{10} = 12.65 \approx 4\pi = 12.56$, so the *super-importance of hue difference* holds for the H2SI colour space.

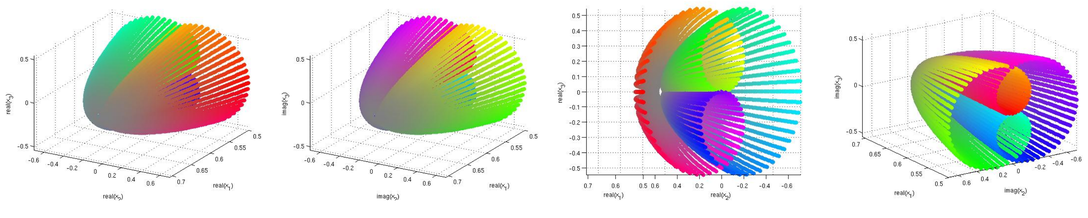


GEODESICS AND PROJECTIONS

The figure to the right shows geodesic lines (shortest distance) in the equatorial plane ($I = \frac{1}{2}$). Similar lines exist for all planes of constant I. For hue differences of over 60° (e.g. red-yellow) the lines leave the plane, as (nearly) predicted by Judd (∼1970).



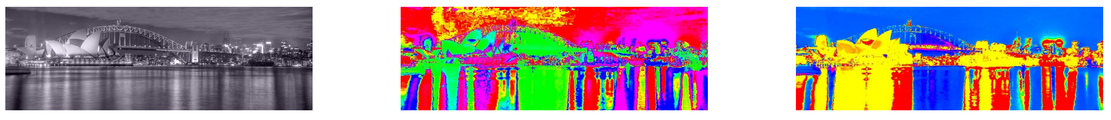
The figures below show some projections of H2SI:



APPLICATIONS

The H2SI representation gives many benefits, such as linear transformations, and that other vector space operations may be applied. E.g. the 'Eigen'-colour decomposition (actually the singular value decomposition).





Applications include image segmentation and compression.



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[3] E. Schrödinger, "Farbenmetrik", Zeitschrift fr̈ Physik A Hadrons and Nuclei, vol. 1, pp. 459–466, 1920. [Online]. Available: http://dx.doi.org/10.1007/BF01332674

[4] L. Silberstein, "Investigations on the intrinsic properties of the color domain", J. Opt. Soc. Am., vol. 28, no. 3, pp. 63–85, Mar. 1938. [Online]. Available: http://www.opticsinfobase.org/abstract.cfm?URI=josa-28-3-63